Math 466 Math of Machine Learning, Spring 2024

Class: WF 3:05PM – 4:20PM @ Biological Science 130
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Course Overview

This course introduces commonly used machine learning techniques with a focus on understanding them mathematically, utilizing tools from calculus, linear algebra, analysis, and probability.

Learning Objectives

By the end of this course, students are expected to:

- (General) know the common concepts, techniques and problems in machine learning
- (Supervised learning) know and understand the mechanisms of linear, ridge, LASSO regression, logistic regression, support vector machine and k-means classification
- (Unsupervised learning) know and understand the mechanisms of PCA, MDS, k-means clustering, isomap, spectral methods, diffusion map and t-SNE
- (Optimization) know whether a given optimization problem has solution or not, is convex or non-convex and how to characterize the optimality conditions of an optimization problem
- (Optimization) know how to find an approximate solution of an optimization model by applying certain optimization algorithm
- (Deep learning) know the basics of fully-connected neural network and convolutional neural network
- (Statistical learning) know why and how to develop concentration inequality, VC dimensions

Textbooks

There are no required textbooks. As necessary, links to papers and other reading materials will be provided. Some general references are:

[HTF09] The Elements of Statistical Learning: Data Mining, Inference, and Prediction, Second Edition, by Trevor Hastie, Robert Tibshirani, and Jerome Friedman. Springer (2009). ISBN 9780387848570.

[V18] High dimensional probability: An Introduction with Applications in Data Science, by Roman Vershynin. Cambridge University Press (2018). ISBN 9781108231596.

[GBC16] Deep Learning, by Ian Goodfellow, Yoshua Bengio, and Aaron Courville. Cambridge: MIT press (2016). ISBN 9780262035613.

[BSS06] Nonlinear programming: theory and algorithms, 3rd edition, by Mokhtar S. Bazaraa, Hanif D. Sherali, and C. M. Shetty. John Wiley & Sons, Inc (2006). ISBN 9780471486008.

[NW06] Numerical optimization, by Jorge Nocedal and Stephen J. Wright. Springer New York (2006). ISBN 9780387303031

Prerequisites

Math: multivariable calculus, linear algebra, and probability.
We will discuss most of the necessary mathematical techniques as we proceed through the semester. But basic knowledge on these three courses is needed to understand the lecture.

Coding: at least one programming language, python or MATLAB are preferred.
There will be programming questions in homework and/or reports. The instructor and TA/grader are not responsible for helping debugging students' programming.

Attendance

Attendance in class is a vital part of the learning process. Regular class attendance is strongly encouraged. It is the student's responsibility to keep informed of any announcements, syllabus adjustments, or policy changes made during scheduled classes.

Grading Policy

Letter grades will be computed from the semester average. Maximum lower bound cutoffs for A, B, C and D grades are 93%, 83%, 73% and 60% respectively. These bounds may be moved lower at the instructor's discretion.

- 4 Homework (40%)

Homework will be assigned by topic. Depending on the content of the lecture, the homework assignment may have a programming part, in which case both codes and a written summary of experimental results (preferably in latex/markdown or any typed-up format) are required. Homework is graded both for accuracy, clarity, and completeness. For some questions, e.g., those that are broken down into branches, partial credit may be granted depending on the quality and completeness of the handed-in solution. Efforts to give meaningful partial solutions are always encouraged, but the grader has the right to determine the points to be credited for the handed-in solutions.

- 1 midterm exam and 1 in-class final exam (15%+15%)

We will have two in-class 75 minutes written exam. The written exams will be closed book and no use of computational aids is allowed.

- Course project (30%)

The course project can be either (i) a review report on a selected topic, or (ii) a technical report on a small research project of machine learning. You can select a topic related to the content of the class.

(i) For the review report, you will summarize the main results in the field of study, organize and cite the related literature, and present the content in a clear, correct, and organized way.

(ii) For the technical report, you will design the content of your project, and present your results (theoretical, programming, or both) in the report.

For either type of report, you will also prepare one slide for a lightning presentation in class. The evaluation will be based on topic selection (2%), proposal (5%), the in-class presentation (8%) and the final written report (15%). We will discuss and provide more guidance in the class about the course report.

- Collaboration policy

  Group work and collaborative efforts for homework and projects are encouraged in this course. However, each handed-in problem set, and report must be independent work. You should name the students or other people with whom you had significant discussions about the problems, if any, on your hand-in solutions for homework. You should present a complete written solution/code to each problem, in your own words, without reference to the written solution of any other person. Any written sources, such as books and online sources other than the course textbook, that contribute significantly to your understanding of the problem should also be cited. Homework and report credits will not be given in case of violating the policies.

- Late policy

  Students have two free “late” days they can use on homework throughout the semester, with *at most one used for any one homework*. A late day is defined as any whole or partial day after the submission deadline. These free late days are to be used for minor illnesses, balancing other course work, conflicts associated with traveling, problems with your computer, etc. After using up the two free late days, late homework will be penalized 10% per business day, and no credit will be awarded once solutions are posted (which can be as soon as the next class). Homework submitted on the due date but after the time specified will be penalized 5%. Late submissions of project reports may result in a complete forfeiture of credit for the report section.

- AI tools policy

  Students are encouraged to explore AI tools in homework and projects while maintaining academic integrity. The use must be properly documented and credited. For example, text or answers generated using ChatGPT-3 should include a citation such as: “Chat-GPT-3. (YYYY, Month DD of query). “Text of your query.” Generated using OpenAI. https://chat.openai.com/” Material generated using other tools should follow a similar citation convention. Please be aware that AI does not always give the right answer or properly cite the references. Students using AI tools have the responsibility to make sure their final hand in homework and project report do not violate the academic integrity and include students’ independent understanding and intellectual contributions.

Academic Integrity

Students are responsible for informing themselves of Duke’s policies regarding academic integrity. Students found in violation of the code are subject to penalties ranging from loss of credit for work involved to a grade of F in the course, and possible risk of suspension or probation. The academic integrity policy will be enforced in all areas of the course, including homework, exams, and projects. If you have any questions concerning this policy before submitting homework or project reports, please ask for clarification.

Topic and Schedule (Tentative)

Introduction
Supervised learning
- Linear, ridge and LASSO regression
- Logistic regression, support vector machine, k nearest neighborhood and decision tree

Optimization
- Convex optimization
- Selected optimization algorithms
  - Gradient, subgradient, proximal gradient methods
  - Stochastic gradient descent

Unsupervised learning
- k means, PCA, MDS
- Manifold learning (isomap, spectral methods, diffusion map, t-SNE)

Concentration inequality, VC dimensions
Deep learning
- Fully connected layers and FCNN
- Convolutional layers and CNN, GNN
- Back propagation
- Universal approximation theorem
- Deep generative models

Reinforcement learning (if time permit)

module 1

week 1, 1/12
introduction

week2, 1/17, 1/19
supervised learning

week3, 1/24, 1/26
supervised learning
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module 2

week4, 1/31, 2/2
2/1 hw1 due
optimization

week5, 2/7, 2/9
optimization

week6, 2/14, 2/16
optimization
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module 3

week7, 2/21, 2/23
2/23 last day for reporting midsemester grades
2/23 hw2 due
unsupervised learning

week8, 2/28, 3/1
2/29 project topic selection due
unsupervised learning

week9, 3/6, 3/8
3/8 mid-term 1
unsupervised learning

3/11-3/15 spring break
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module 4

week10, 3/20, 3/22
3/21 hw3 due
statistical learning

week11, 3/27, 3/29
3/27 last day for withdraw with W
3/28 project proposal/initial report due
statistical learning
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module 5

week12, 4/3, 4/5
4/4 hw4 due
deep learning

week13, 4/10, 4/12
deep learning

week14, 4/17, 4/19
4/17,19 lightning talk

week15, 4/24
4/24 in-class final

4/18-28, graduate reading period
4/25-28, undergraduate reading period
4/30, Project report due

*Note: This syllabus is subject to change as the semester progresses.